

## CHAPTER 2

### Approach to the problem

The principal approach to the problem is to compare and hence match the performances of a pulse-coded neuron (PCN) based network (PCNN, pulse-coded neural network) with an adaptive network. There is currently no physiologically plausible algorithm for an adaptive pulse-coded neural network (APCNN). Hence two models of different representations (phenomenological and physiological) were taken. A model developed by Grossberg was chosen for the phenomenological representation. For the physiological representation, a network comprised of PCN's was developed using Grossberg's method of minimal anatomies [Grossberg 1971, 1972a]. Grossberg's network is inherently adaptive while the PCNN is not. Comparing performances of the two networks using Grossberg's network as the reference, parameters within the PCNN may be tuned to achieve an APCN. This approach belongs to the class of adaptation methods known as model reference adaptation. It is conjectured that success in achieving a model reference APCNN could lay the groundwork for future development of PCNN adaptation by actor-critic methods [Werbos 1992].

### Grossberg's Dipole Network

The phenomenological representation of brain function developed by Grossberg (Fig. 2.1) grossly models the scale of functional magnetic resonance imaging (fMRI) activity. This model was first introduced by Grossberg in a 1972 paper [Grossberg 1972b]. The model, due to its ability to either exhibit excitatory or inhibitory signal output depending upon the input can be considered a dipole network (G-DN, Grossberg's Dipole Network).

It should be noted that though the G-DN has nodes with connections resembling neuron to neuron connectivity, the activities from each node and synaptic weights represent a collective/population outcome. Figure 2.2 shows the G-DN implemented in this thesis. The G-DN used in the thesis is the same but with some change in notation and language. For instance, tonic and shock inputs are called bias and drive input respectively for a more general application of the input terminology, i.e., bias and drive inputs for both G-DN and PCNN.

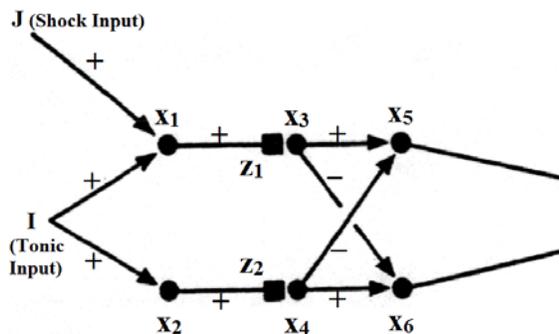


Figure 2.1. Dipole network of 6 nodes adopted from Grossberg’s 1972 paper [Grossberg 1972b] receiving tonic (I) and shock (J) inputs. All the arrows represent excitatory (+) connection except for cross-inhibitory connection arrows from 3<sup>rd</sup> to 6<sup>th</sup> node and 4<sup>th</sup> to 5<sup>th</sup> node.  $x_i$  represents the activity of  $i^{\text{th}}$  node while  $z_i$  represent weights (elastic weights) for the respective elastic connections ( $x_1$  to  $x_3$  &  $x_2$  to  $x_4$ ). Note that for the elastic connection the arrow-heads are replaced by boxes. For the network to be a dipole network one of the last two node outputs (either  $x_5$  or  $x_6$ ) is excitatory while the other is inhibitory.

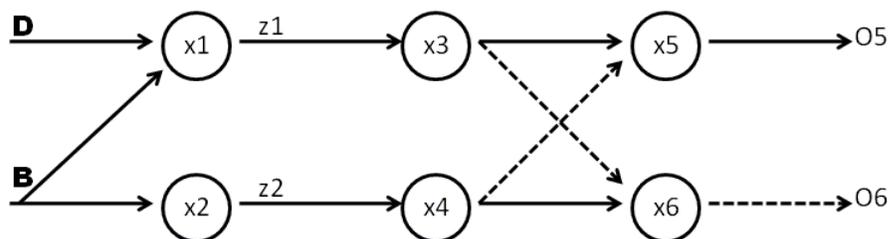


Figure 2.2. Dipole network used. The network remains same as in Figure 2.1 with minor change in description convention. Tonic input (I) is replaced by bias (B) and shock input (J) by drive (D). Solid lines represent excitatory connection while dashed lines represent inhibition. For the G-DN used in the thesis, node five activity ( $x_5$ ) leads to excitatory output ( $O_5$ ) while  $x_6$  leads to inhibitory output ( $O_6$ ).

The equations for the G-DN (Fig. 2.2) are,

$$\dot{x}_1 = -\alpha \cdot x_1 + B + D, \quad (1)$$

$$\dot{x}_2 = -\alpha \cdot x_2 + B, \quad (2)$$

$$\dot{z}_1 = \beta \cdot (\gamma - z_1) - \delta \cdot [x_1(t - \tau) - \Gamma]^+ \cdot z_1, \quad (3)$$

$$\dot{z}_2 = \beta \cdot (\gamma - z_2) - \delta \cdot [x_2(t - \tau) - \Gamma]^+ \cdot z_2, \quad (4)$$

$$\dot{x}_3 = -\varepsilon \cdot x_3 + \zeta \cdot [x_1(t - \tau) - \Gamma]^+ \cdot z_1, \quad (5)$$

$$\dot{x}_4 = -\varepsilon \cdot x_4 + \zeta \cdot [x_2(t - \tau) - \Gamma]^+ \cdot z_2, \quad (6)$$

$$\dot{x}_5 = -\omega \cdot x_5 + \kappa \cdot [x_3(t - \sigma) - x_4(t - \sigma)]^+, \quad (7)$$

$$\dot{x}_6 = -\omega \cdot x_6 + \kappa \cdot [x_4(t - \sigma) - x_3(t - \sigma)]^+, \quad (8)$$

$$O_5 = \lambda \cdot [x_5 - \Omega]^+, \quad (9)$$

$$O_6 = \lambda \cdot [x_6 - \Omega]^+. \quad (10)$$

with parameters;  $\alpha = 3$ ,  $\beta = 1$ ,  $\delta = 2/3$ ,  $\gamma = 3$ ,  $\varepsilon = \omega = 4$ ,  $\zeta = 4/3$ ,  $\kappa = 1$ ,  $\lambda = 32$ ,  $\Gamma = 1/2$  and  $\Omega = 0$ . Stimulus B (bias) is either 2 or 0 and D (drive) either 1 or 0. The model was simulated (Matlab®) using Euler's method for the difference equation given above with an increment  $\Delta t$  of 0.01, hence  $\Delta t = t - \tau = t - \sigma$ . The Heaviside extractor activation

$$\text{function } [H]^+ \text{ is given by, } [H]^+ = \begin{cases} H, & \text{if } H > 0 \\ 0, & \text{if } H \leq 0. \end{cases}$$

The network G-DN receiving only bias (B) input (tonic input) gives no output due to the equal strength of excitatory and inhibitory (cross-inhibition) inputs into the last two

nodes, five and six (Eqns. 7 & 8). However with the addition of drive (D) input into one of the first two nodes (node-1 in Fig. 2.2) this balance is altered. That is, node-4 with lesser activity (than node-3) sends an inhibitory output into node-5 that is weaker than the excitatory input from node-3. This is reversed for node-6. Hence node-5 shows activities during dual (B & D) stimuli while there is no activity in node-6. But as soon as the D-stimulus is removed and the network reverts back to receiving only B-stimulus, activity in node-6 occurs for a short duration before the last two nodes (node-5 & node-6) have no activities due to above reasons (mutual inhibition). This is known as rebound phenomenon caused by the elastic  $z_i$  weights (Eqns. 3 & 4), a short-term memory-like effect. The above described behavior of the dipole network is shown in Figures 2.3 and 2.4.

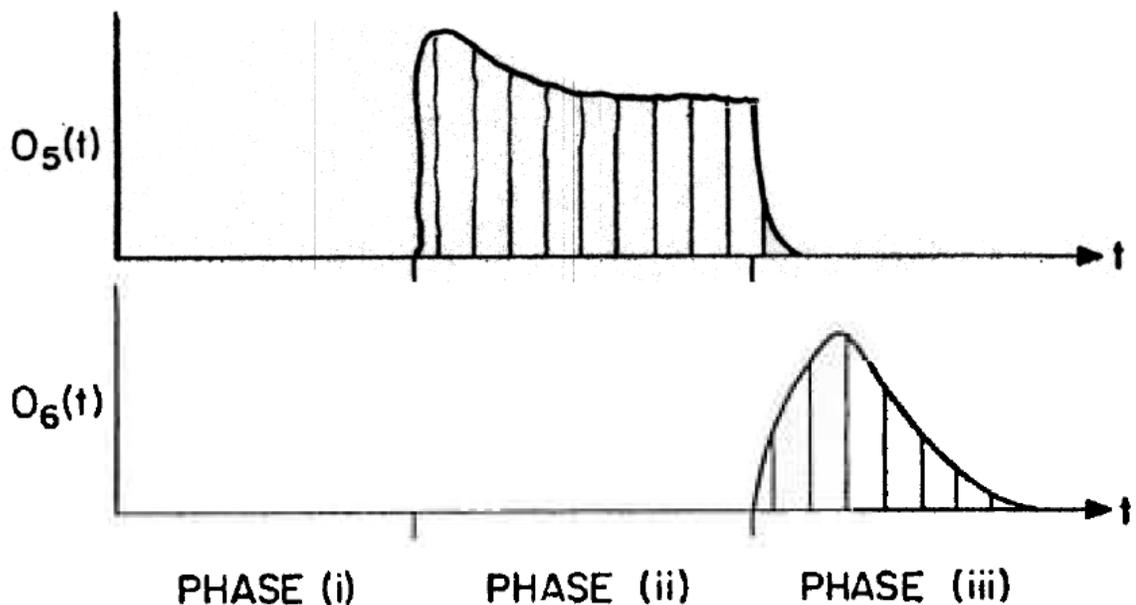


Figure 2.3. Persistent suppression and transient positive/rebound response to shock as seen in  $O_6$  activity trace, following Grossberg's analysis of equations (1 to 10). Phase (i) & (iii) are when network receives only tonic (or bias) input while phase (ii) has the addition shock (or drive) input. The figure is taken from Grossberg's 1972 paper [Grossberg 1972b].

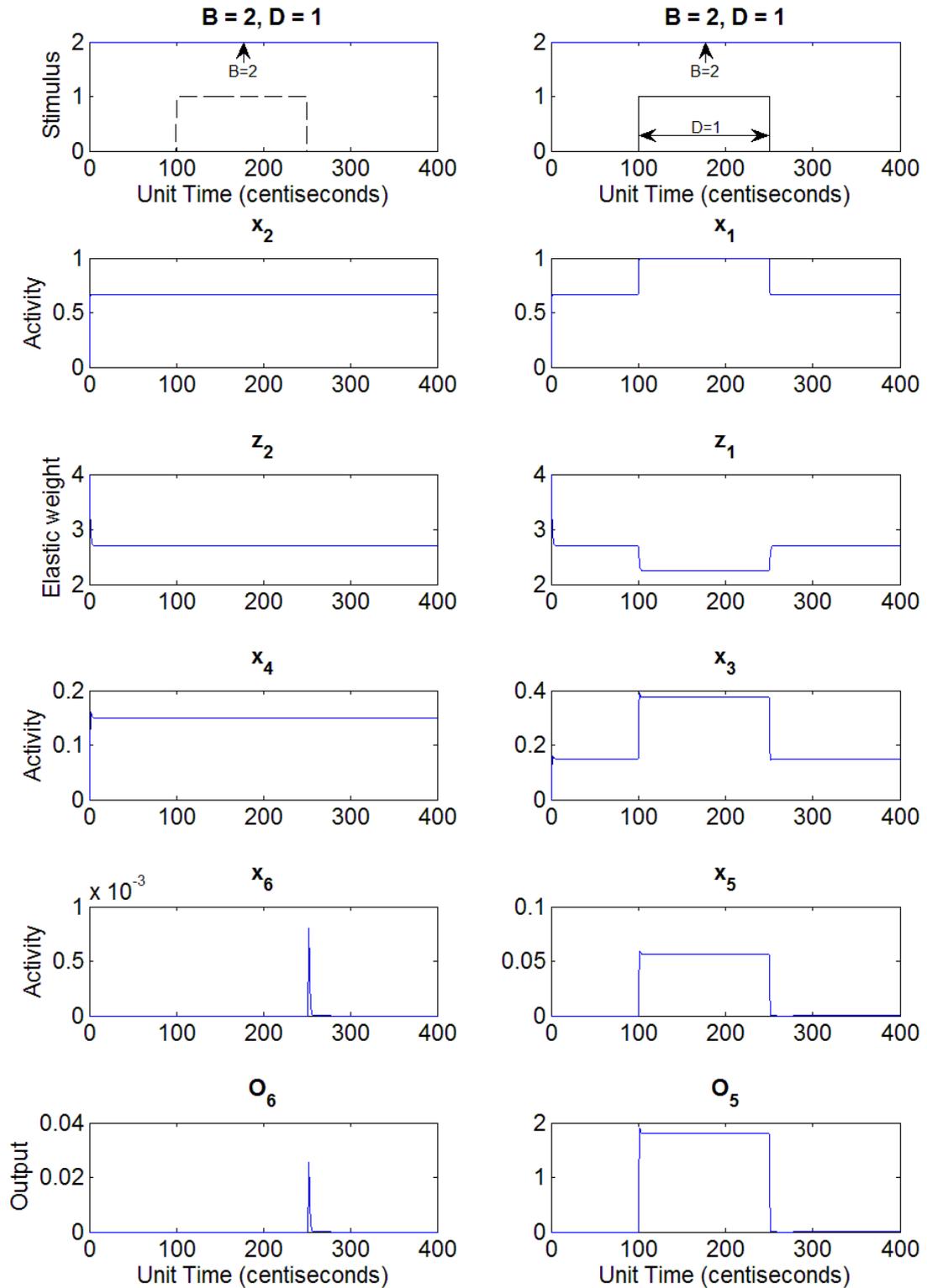


Figure 2.4. Simulation (time-step,  $\Delta t = 0.01$ ) of G-DN from equations (1 to 10) shows the persistent suppression & rebound response seen in Figure 2.3.

### Adaptation in Grossberg's Network

The dipole network (G-DN) with an additional sensory input sends outputs to the motor (M) node. This constitutes the Grossberg network (G-N) as shown in Figure 2.5. The adaptation of the network (G-N) takes place by adjusting the connection strengths (weights) between sensory input and nodes within the G-DN.

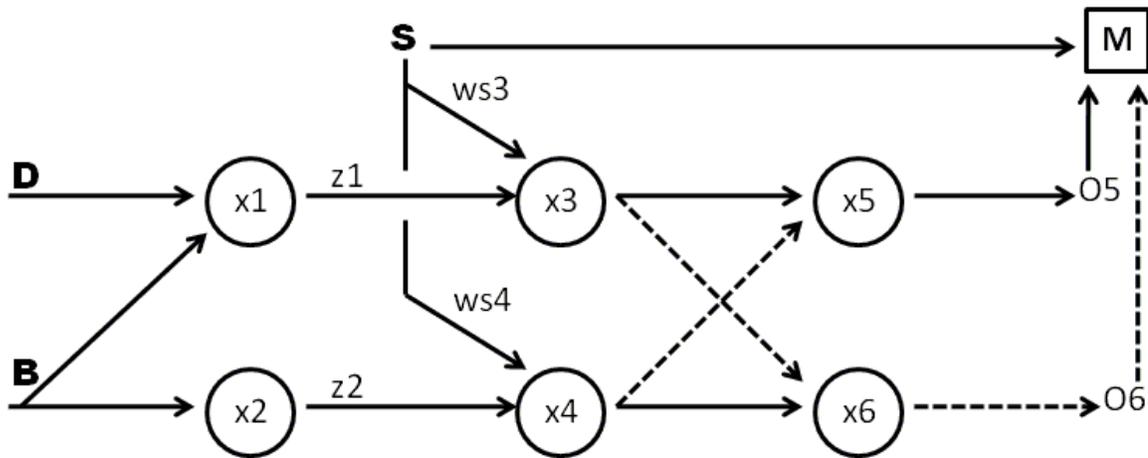


Figure 2.5. G-DN with sensory (S)/conditioning stimulus connected to x3 & x4 by respective synaptic weights ( $w_{s3}$  &  $w_{s4}$ ) which are adaptive. The outputs from the dipole network in addition to the sensory stimulus are the inputs for the motor node (M). This configuration is called the Grossberg network, G-N. (Solid line: excitatory & Dashed line: inhibitory connection).

Activity of the M-node is a reflection of unconditioned response (UR) with unconditioned stimulus (US, Drive: D) and conditioned responses (CR) with conditioned stimulus (CS, post-learned Sensory: S).

The mathematical description of the G-DN was based on functionals [Grossberg 1972c] rather than functions. That is, the original G-DN did not present a long-term adaptive function. However, recent works by Hill and Wells [Hill et al. 2009] have shown that incorporating Grossberg's outstar rule based on Hebb's principle [Hebb 1949] can make the G-DN stably adaptive. However their rule required some alterations for the network used in the thesis. The modified version of the rule implemented in G-N is

$$\dot{w}_{ji} = -\nu_{ji} \cdot w_{ji} + \eta \cdot [s_j - \Gamma_s]^+ \cdot [x_i - \Gamma_o]^+, \quad (11)$$

$$\dot{\nu}_{ji} = -\varphi \cdot \nu_{ji} + \rho \cdot [s_j - \Gamma_\nu]^+ + \rho' \cdot [x_h - \Gamma_{\nu'}]^+ \cdot (s_j - \Gamma_\nu)_0^1, \quad (12)$$

with parameters;  $\eta = 4.4$ ,  $\Gamma_s = 1/2$ ,  $\Gamma_o = 0.35$ ,  $\Gamma_\nu = 0.79$  and  $\Gamma_{\nu'} = 0.67$ . The notation is such that  $x_i$  is the activity of the node whose synaptic connection with the sensory node is being adapted and  $x_h$  is the activity of the preceding node within the G-DN elastically connected to  $i^{\text{th}}$  node. Sensory input  $s_j$  is either 0.8 or 0 and the weight values  $w_{ji}$  are constrained such that  $0 \leq w_{ji} \leq 0.5$ . Note that presence of the very fast function,  $\dot{\nu}_{ji}$  causes  $\dot{w}_{ji}$  to retain its values (i.e., learns). For reasons of computation,  $\nu_{ji}$  being a very fast function simulation was done by considering  $\dot{\nu}_{ji} = 0$ , i.e.,  $\nu_{ji}$  reaches steady-state very fast. The equation used in the simulation instead of equation (12) is,

$$\nu_{ji} = c \cdot [s_j - \Gamma_\nu]^+ + c' \cdot [x_h - \Gamma_{\nu'}]^+ \cdot (s_j - \Gamma_\nu)_0^1 \quad (13)$$

with parameters,  $c = 0.03$  and  $c' = 1$  with Heaviside step function,  $(S)_0^1 = \begin{cases} 1, & \text{if } S > 0 \\ 0, & \text{if } S \leq 0 \end{cases}$ .

In the Grossberg's network implemented for the thesis, the middle nodes (node-3 & node-4) receive sensory inputs (Fig. 2.5) whose synaptic connections are determined by equations (11) and (13). Thus,  $j = s$  and  $i \in \{3, 4\}$ , implying each node (node-3 & node-4) have equations (11) and (13). This means that equations (5) and (6) are replaced by

$$\dot{x}_3 = -\varepsilon \cdot x_3 + \zeta \cdot [x_1(t - \tau) - \Gamma]^+ \cdot z_1 + \sum_{j=1}^n w_{j3} \cdot s_j, \quad (5')$$

$$\dot{x}_4 = -\varepsilon \cdot x_4 + \zeta \cdot [x_2(t - \tau) - \Gamma]^+ \cdot z_2 + \sum_{j=1}^n w_{j4} \cdot s_j. \quad (6')$$

Finally, the motor (M) node receives inputs from the sensory and dipole network outputs. This is expressed as,

$$M = \mu \cdot [(s_j + O_5 - O_6) - \Xi]^+, \quad (14)$$

with parameters;  $\mu = 1$  and  $\Xi = 1$ . Note that the overall behavior of the G-N network is defined by the M-node activity. That is, M-node activity occurring during B and D-stimulus is called the unconditioned response to unconditioned stimulus (D-stimulus) while M-node activity during B and S-stimulus after conditioning (learning) is called conditioned response to condition stimulus (S-stimulus).

Simulation of the G-N shows the long-term adaptive property (or just adaptive property) of the network (Fig. 2.6, 2.7 & 2.8). Immediately after the learning phase (when all B, D and S-stimulus are given) the network remains learned, although there is a small decrease in adapted weight value ( $\approx 0.3\%$  of peak). It should be noted that learning occurs during simultaneous conditioning and delay conditioning paradigms [Balkenius 1995] but not with trace or backward conditioning. This is because the connection assumed in the G-N follows Hebb's cell-assembly theory [Hebb 1949].

Finally, Figure 2.9 shows that the behavior produced (M-node activity to S-stimulus) and the association between condition and unconditioned stimulus eventually gets extinguished. This overall adaptive property of the G-D network can then be taken as a reference to achieve an adaptive PCNN (APCNN) that is psycho-neurologically feasible.

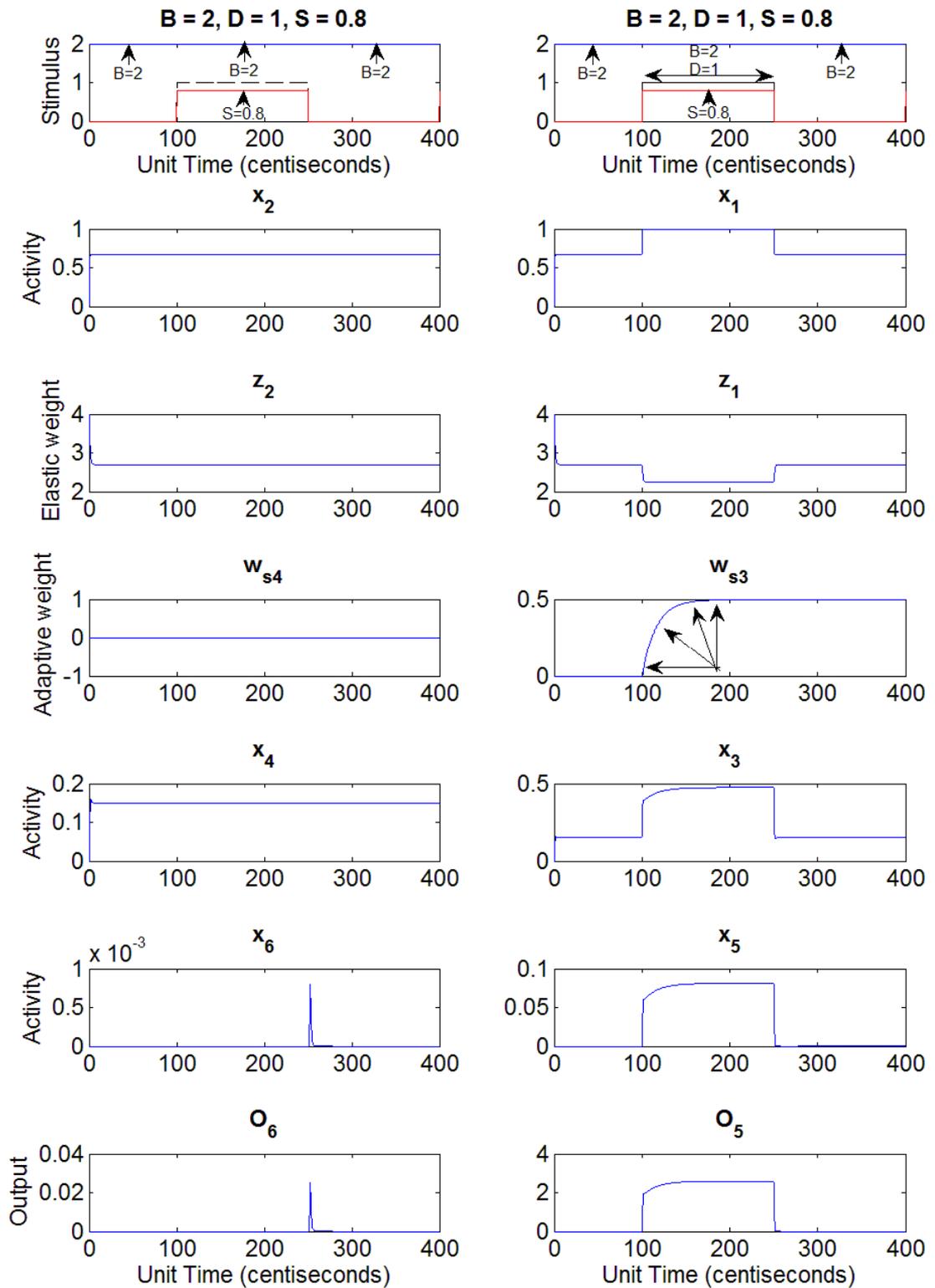


Figure 2.6. Simulation (time-step,  $\Delta t = 0.01$ ) of G-N (Eqns:1 to 4, 5', 6', 7 to 11 & 13) shows learning (cartwheel arrows) in the network, i.e., rise of  $w_{s3}$  strengthening the connection in part of network during association of S to D stimulus.

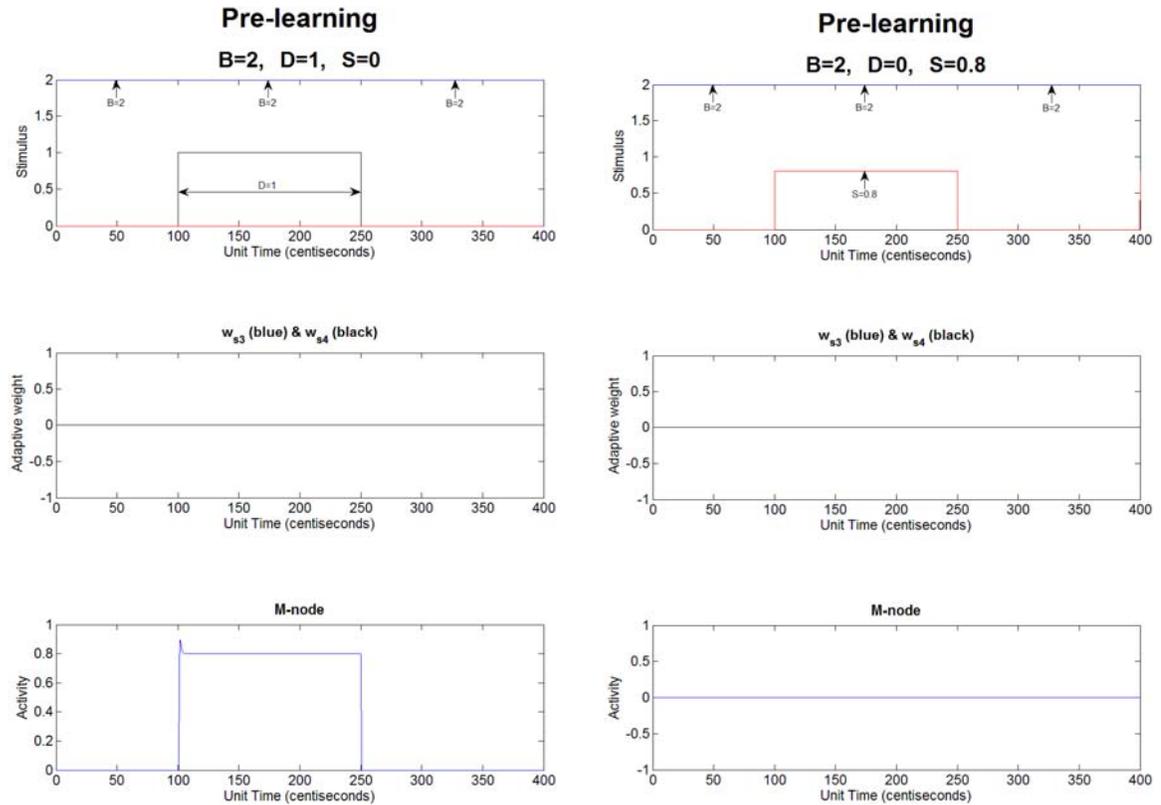


Figure 2.7. Simulation (time-step,  $\Delta t = 0.01$ ) of G-N (Eqns: 1 to 4, 5', 6', 7 to 11 & 13) shows network behavior prior to learning/conditioning.

Left: M-node activity (bottom) during B & D stimulus (top) representing unconditioned response to D-stimulus. No learning takes place (middle).

Right: prior to conditioning and hence before association process between conditioning (S) stimulus and unconditioned (D) stimulus there is no M-node activity (bottom) with S-stimulus.

In both cases  $w_{s3}$  and  $w_{s4}$  remains zero, i.e., no learning takes place and hence blue ( $w_{s3}$ ) and black ( $w_{s4}$ ) values are overlapping.

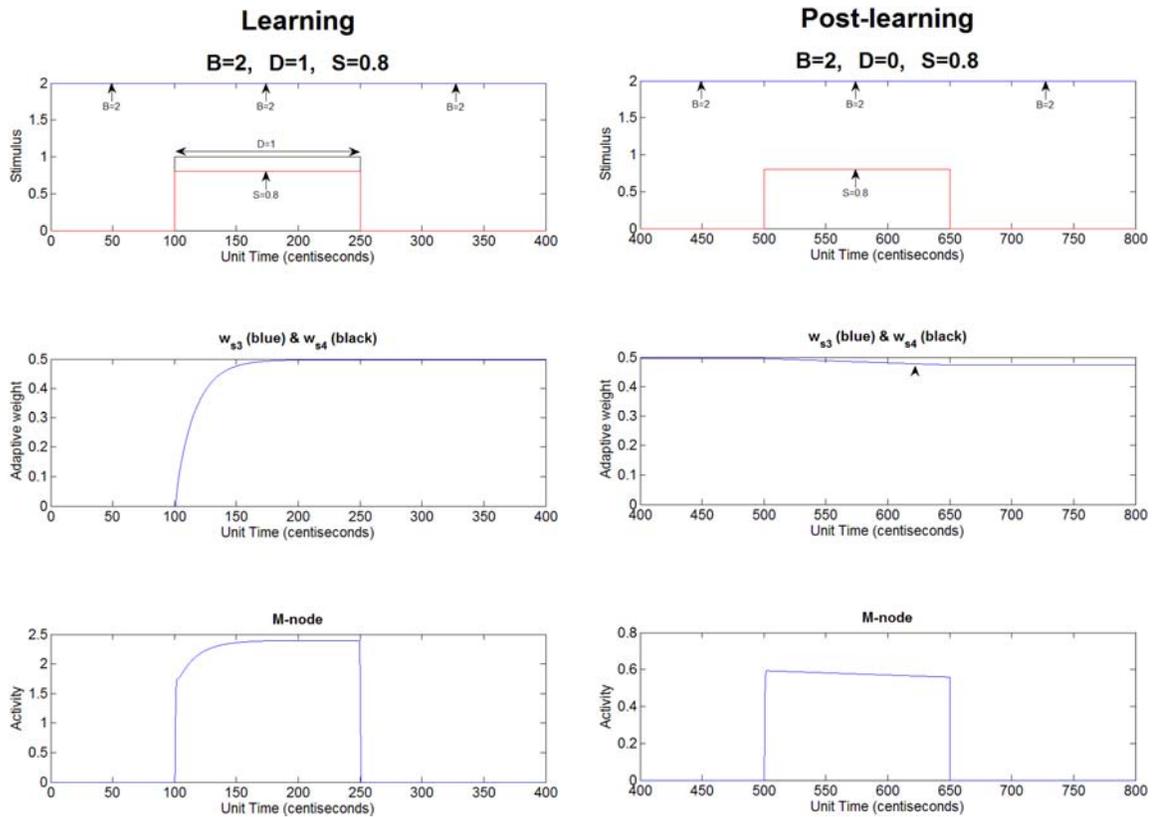


Figure 2.8. Simulation (time-step,  $\Delta t = 0.01$ ) of G-N (Eqns: 1 to 4, 5', 6', 7 to 11 & 13) shows network behavior after learning/conditioning.

Left: M-node activity (bottom) during B, D & S stimulus (top) representing response during the association process of conditioning (S) stimulus to unconditioned (D) stimulus. Learning occurs during conditioning (middle).

Right: after conditioning the conditioned (S) stimulus is capable to inducing a conditioned response (bottom).

During learning (left),  $w_{s4}$  remains zero while  $w_{s3}$  (blue) connected with the G-DN node (node-3) at the receiving end of all three stimuli (B, D & S) rises.

Post-learning (right) during the duration of condition stimulus (S),  $w_{s3}$  (blue) has a very small decrease (arrowhead, middle) from its plateau achieved during learning. This is the consequence of the G-N's use of a Hebbian-class learning rule.

## Conditioning & Extinction

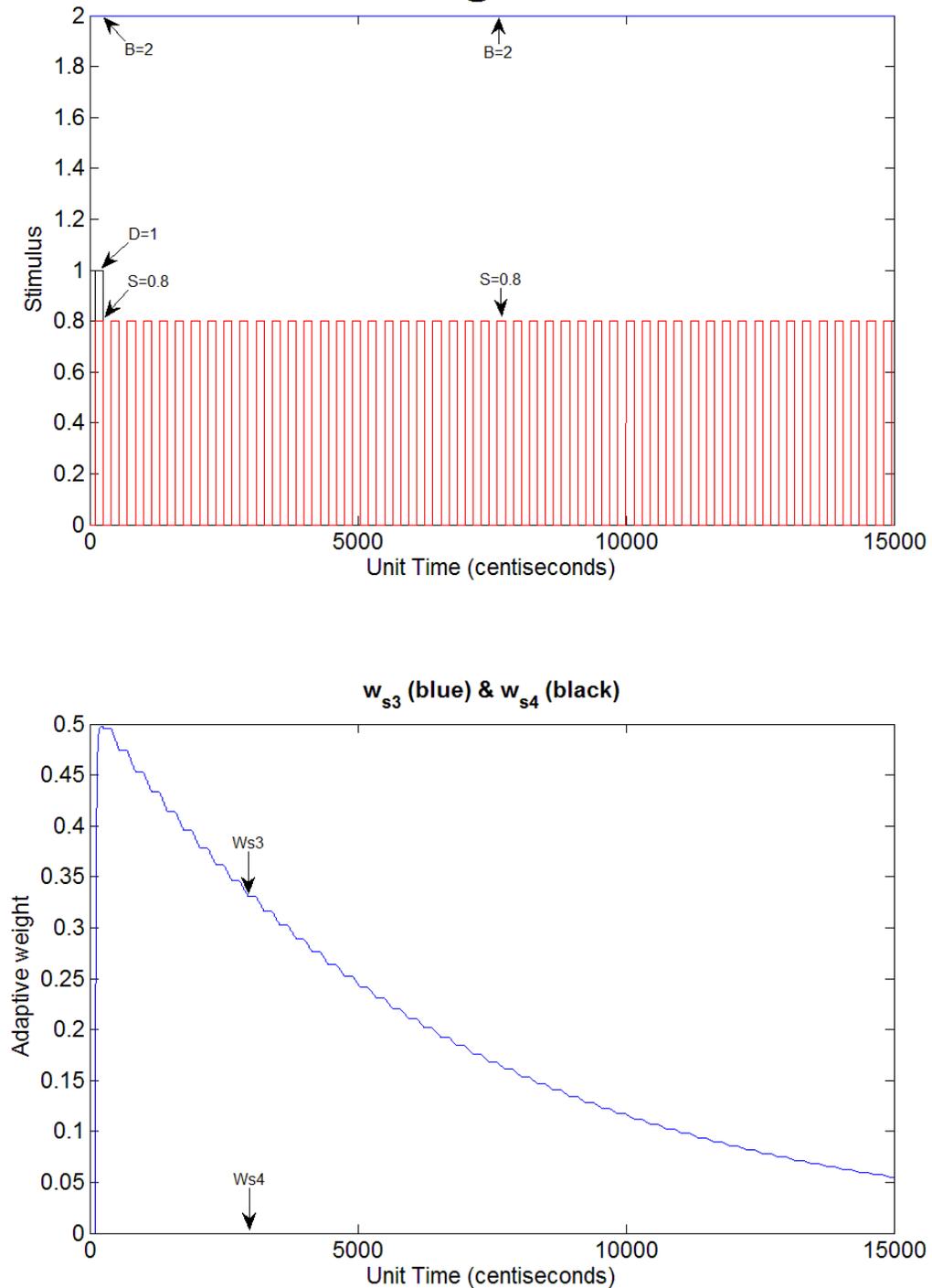


Figure 2.9. Simulation (time-step,  $\Delta t = 0.01$ ) of G-N (Eqns:1 to 4, 5', 6', 7 to 11 & 13) during extinction experiment. First phase ( $B = 2, D = 1, S = 0.8$ ): establishment of association or learning or conditioning as seen in Figure 2.8 (left). Second phase (intervals of  $B = 2, S = 0.8$ ): only S (conditioned) stimulus is applied (without unconditioned D-stimulus) extinguishing the developed association ( $\downarrow w_{s3}$ ) with each successive S-stimulus.

### Building a PCNN using Eckhorn's model (ENU)

The functional unit used for the PCNN analogue of the G-N is the Eckhorn neuron unit (ENU). The ENU model introduced by Eckhorn et al. [Eckhorn et al. 1990] has been shown to be a population neuron model [Wells et al. 2006]. Figures 1.5, 1.6 and 1.7 show the components of a ( $i^{\text{th}}$ ) basic ENU whose equations are

$$FF_i(t) = FF_i(t - \Delta t) \cdot \exp(-\Delta t / \tau_{ff}) + (1 / \tau_{ff}) \cdot w_{ff} \cdot \sum_{\forall j} F_j^{inputs}, \quad (15)$$

$$LF_i(t) = LF_i(t - \Delta t) \cdot \exp(-\Delta t / \tau_{lf}) + (1 / \tau_{lf}) \cdot w_{lf} \cdot \sum_{\forall j} L_j^{inputs}, \quad (16)$$

$$U_i(t) = FF_i(t) \cdot (1 + LF_i(t)), \quad (17)$$

$$V_i(t) = \sum_{\forall i} U_i(t), \quad (18)$$

$$\Theta_{V_i}(t + \Delta t) = \begin{cases} V_{pg}, & V_i(t) \geq \Theta_i(t) \\ \Theta_{V_i}(t) \cdot \exp(-\Delta t / \tau_{pg}), & V_i(t) < \Theta_i(t) \end{cases}, \quad (19)$$

$$\Theta_i(t) = \Theta_O + \Theta_{V_i}(t), \quad (20)$$

$$Z_i(t + \Delta t) = \begin{cases} 1, & V_i(t) \geq \Theta_i(t) \\ 0, & V_i(t) < \Theta_i(t) \end{cases}. \quad (21)$$

Equations (15), (16) and (17) represents dendrite component. The dendrite outputs are summed to become soma input (Eqn. 18). The remaining equations (Eqn. 19, 20 & 21) represent the soma component. The time-step ( $\Delta t = 1$ ) for simulation of PCNN and parameter value are given in the next chapter.

The Eckhorn model (ENU) was chosen over other proxy neuron models particularly the integrate and fire (I&F) model [Burkitt 2006a, 2006b], because it overcomes the

shortcomings of a basic I&F model namely, presence of refractory period (Eqns. 19 & 20), ability to generate bursting response due to the absence of reset functions in an ENU and the capability for synchronous firing without being in all-pass mode (Eqns. 16 & 17). Other models such as Rulkov's model [Rulkov 2002] and the Hodgkin-Huxley model [Hodgkin & Huxley 1952] though biologically more realistic than the ENU were not chosen for developing the PCNN because of the complications in choosing the parametric values or because of greater computational cost. The design process for developing the ENU based network analogue of a G-DN is discussed first and then its behavior in the following two chapters.

### **Incorporation of the adaptive property of G-N to achieve APCNN**

The property of an adaptive G-N can be incorporated into the E-N using the adaptation scheme called performance feedback adaptation [Widrow & Stearns 1985]. In this schema the M-node activity of G-N is taken as the reference performance for the M-node spikes of E-N to achieve (match). That is, the performance reference or desired response is compared with the output of the E-N such that the adjustable variables within the E-N are tuned to achieve an output that closely (minimum error) resembles the desired output (figure 2.10).

Bridging of the two networks (G-N & E-N) requires one of the network outputs to be transformed before comparison between performances. This is because M-node outputs (activities) from G-N represent functional Magnetic Resonance imaging (fMRI) while M-node outputs (spikes) from E-N mimic electro-encephalogram (EEG) readings. Hence different signal types, real signals for G-N and spikes for E-N. With regard to signal analysis the major area where fMRI's and EEG's differ is their temporal resolution.

Temporal resolutions of fMRI's are in seconds while EEG's are in milliseconds [Gazzaniga et al. 2006].

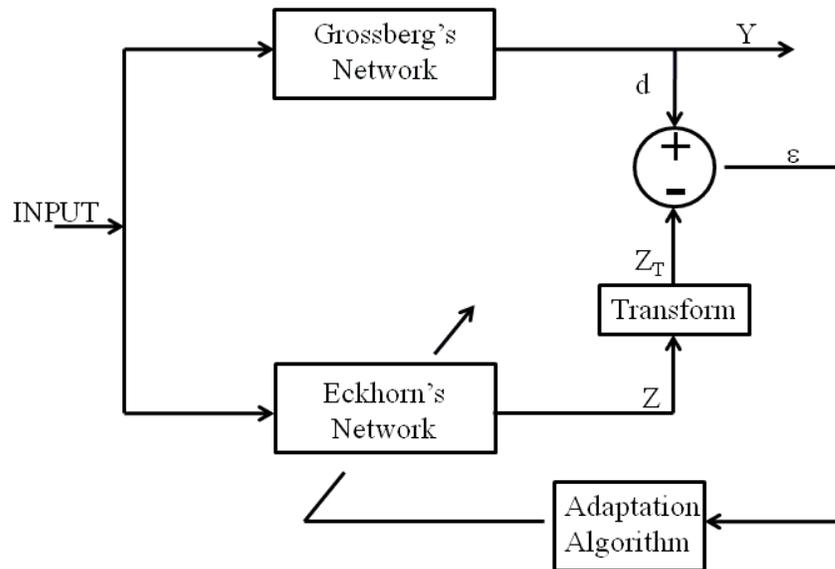
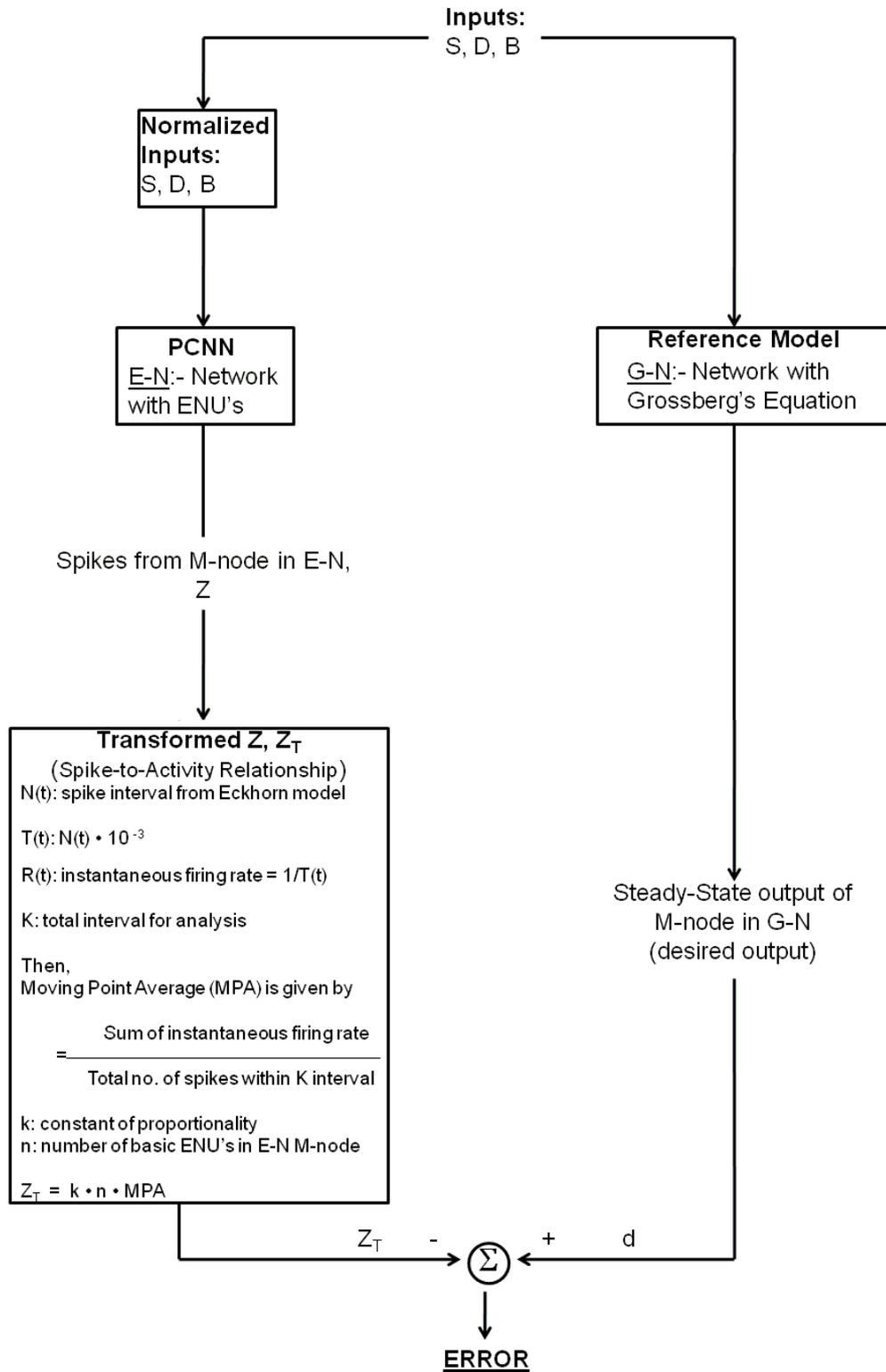


Figure 2.10. Principle approach to the problem. The desired response ( $d$ ) is the output from Grossberg's network while  $Z_T$  is the transformed spike outputs from Eckhorn's. The error between these outputs ( $\epsilon$ ) is then used by the algorithm to adapt the weights.

Since M-node activity of the G-N is the reference or desired output ( $d$ ), the M-node spikes of E-N are transformed as shown in Figure 2.11. This transformation is based on the method of moving point averages (MPA) [Whittaker & Robinson 1967]. For performance evaluation the E-N spike signal is transformed as a product of MPA, an arbitrary constant ( $k=0.5$ ) and the number of basic ENU's ( $n$ ) within the M-node of E-N (i.e.,  $Z_T = k \cdot n \cdot \text{MPA}$ ). Hence the transformed E-N outputs ( $Z_T$ ) can be directly compared with “ $d$ ”.

To our best knowledge, the use of model-reference performance feedback to tune a pulse-coded neural network has not been previously reported. This method is therefore an original contribution of this thesis.



Z<sub>T</sub> -

Σ

+ d

Figure 2.11. Approach implemented for obtaining the error needed for measuring the performance. The desired output (d) is taken as the steady-state (single sample) M-node output of G-N. This is then compared with the transformed spike output (Z<sub>T</sub>) derived by taking the moving point average (MPA).